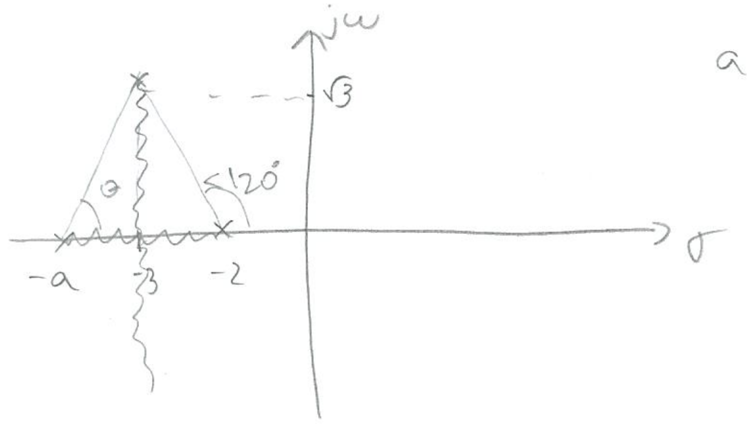
# GABARITO CSM - 2019.1

1) É impossível que o LR passe em $s^*$ pois o LR sempre começa e termina no eixo real

2)



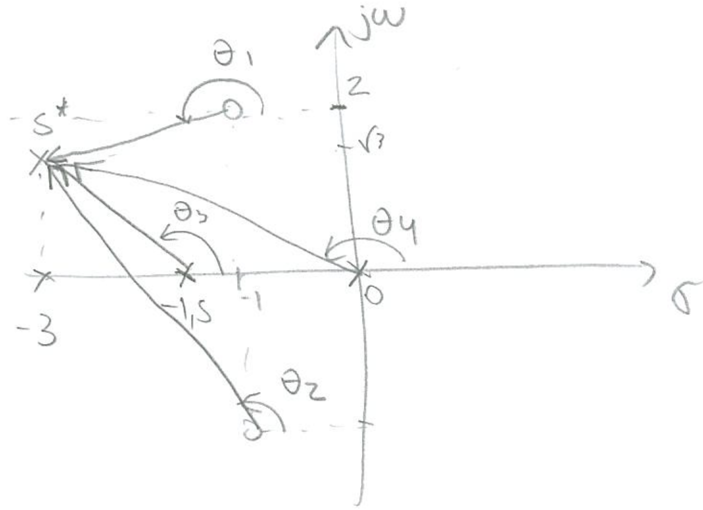

a?

$$-\angle s^*+2 - \angle s^*+a = \pm 180°(2k+1), \; k=0,1,2,\dots$$

$$-\angle -1+j\sqrt{3} - \angle a-3+j\sqrt{3} = -120° - \operatorname{atan}\frac{\sqrt{3}}{a-3} = \pm 180°(2k+1)$$

$$\tan 60 = \sqrt{3} = \frac{\sqrt{3}}{a-3} \Rightarrow \boxed{a=4}$$

3)




$z_1 = 1+j2$

$p_1 = 0$

$p_2 = 1{,}5$

$s^* = -3+j\sqrt{3}$

$$\angle G(s^*)H(s^*) = \angle \frac{(s^*+z_1)(s^*+\bar{z}_1)}{s^*(s^*+3)} = \theta_1+\theta_2-\theta_3-\theta_4 = 24{,}92°$$

$s^*+1+j2 = -2+j(2+\sqrt{3}) = 4{,}2 \angle 118{,}2°$

$s^*+1-j2 = -2+j(\sqrt{3}-2) = 2{,}0 \angle 187{,}6°$

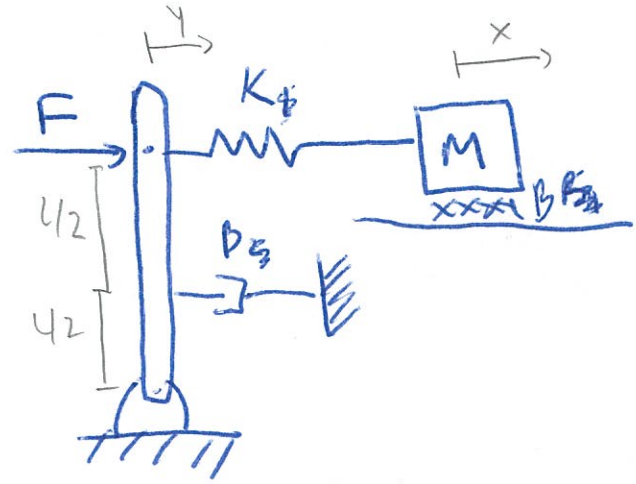
$s^* = -3+j\sqrt{3} = 3{,}5 \angle 150°$

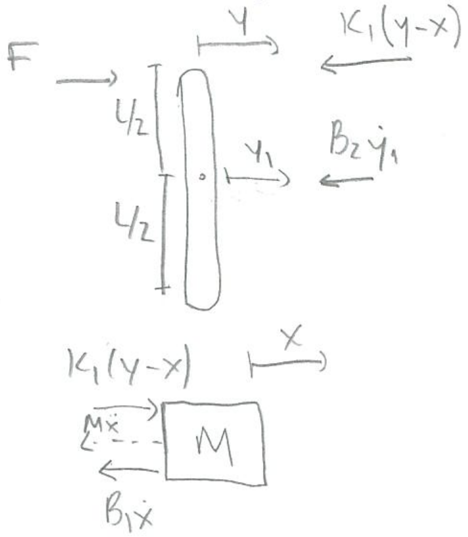
$s^*+1{,}5 = -1{,}5+j\sqrt{3} = 2{,}3 \angle 130{,}9°$

$\phi + \angle G(s^*)H(s^*) = \pm 180(2k+1), \; k=0,1,2,\dots$

$\phi = 180 - 24{,}92 = 155{,}08°$

4)




$$\tan\theta = \frac{y_1}{L/2} = \frac{y}{L} \Rightarrow y_1 = y/2$$

$$\dot{y}_1 = \dot{y}/2$$

$$\frac{L}{2} B_2 \dot{y}_1 + L K_1 (y-x) - LF = 0$$

$$L \frac{B_2 \dot{y}}{4} + L K_1 y - L K_1 x - LF = 0$$

$$(1) \quad B_2 \dot{y} + 4K_1 y - 4K_1 x - 4F = 0$$

$$K_1(y-x) - M\ddot{x} - B_1\dot{x} = 0$$

$$(2) \quad K_1 y - K_1 x - M\ddot{x} - B_1\dot{x} = 0$$

TL em (1)

$$B_2 s Y(s) + 4K_1 Y(s) - 4K_1 X(s) - 4F(s) = 0$$

$$B_2 s Y(s) + 4K_1 Y(s) - \frac{4K_1^2 Y(s)}{M_1 s^2 + B_1 s + K_1} - 4F(s) = 0$$

TL em (2)

$$K_1 Y(s) - K_1 X(s) - M_1 s^2 X(s) - B_1 s X(s) = 0$$

$$X(s) = \frac{K_1}{M_1 s^2 + B_1 s + K_1} Y(s) \quad (\mathrm{I})$$

$$\left[(M_1 s^2 + B_1 s + K_1)(B_2 s + 4K_1) - 4K_1^2\right] Y(s) = 4F(s)(M_1 s^2 + B_1 s + K_1)$$

$$\left(B_2 M_1 s^3 + 4K_1 M_1 s^2 + B_1 B_2 s^2 + 4K_1 B_1 s + B_2 K_1 s + \cancel{4K_1^2} - \cancel{4K_1^2}\right) Y(s) = 4F(s)$$

$$\frac{Y(s)}{F(s)} = \frac{4(M_1 s^2 + B_1 s + K_1)}{B_2 M_1 s^3 + (B_1 B_2 + 4K_1 M_1)s^2 + (4K_1 B_1 + B_2 K_1)s}$$

$$= \frac{\frac{4}{B_2} s^2 + \frac{4B_1}{B_2 M_1} s + \frac{4K_1}{B_2 M_1}}{s\left(s^2 + \frac{B_1 B_2 + 4K_1 M_1}{M_1 B_2} s + \frac{4K_1 B_1 + B_2 K_1}{M_1 B_2}\right)}$$

$B_1 = B_2 = 4$ ; $M_1 = 2$ ; $K_1 = 2$

II $$\frac{Y(s)}{F(s)} = \frac{s^2 + 2s + 1}{s(s^2 + 4s + 5)} = \frac{(s+1)^2}{s(s^2 + 4s + 5)}$$

II → I $$X(s) = \frac{2}{2s^2 + 4s + 2} Y(s) = \frac{1}{s^2 + 2s + 1} Y(s)$$

$$= \frac{F(s)}{s(s^2 + 4s + 5)} \Rightarrow \frac{X(s)}{F(s)} = \frac{1}{s(s^2 + 4s + 5)}$$

$s = -2 \pm j$

OUTRA FORMA DE OBTER A FT :

TL em ①  $B_2 s Y(s) + 4K_1 Y(s) - 4K_1 X(s) - 4F(s) = 0$

TL em ②  $Y(s) = \dfrac{(M_1 s^2 + B_1 s + K_1)}{K_1} X(s)$

① → ②  $(B_2 s + 4K_1) Y(s) - 4K_1 X(s) - 4F(s) = 0$

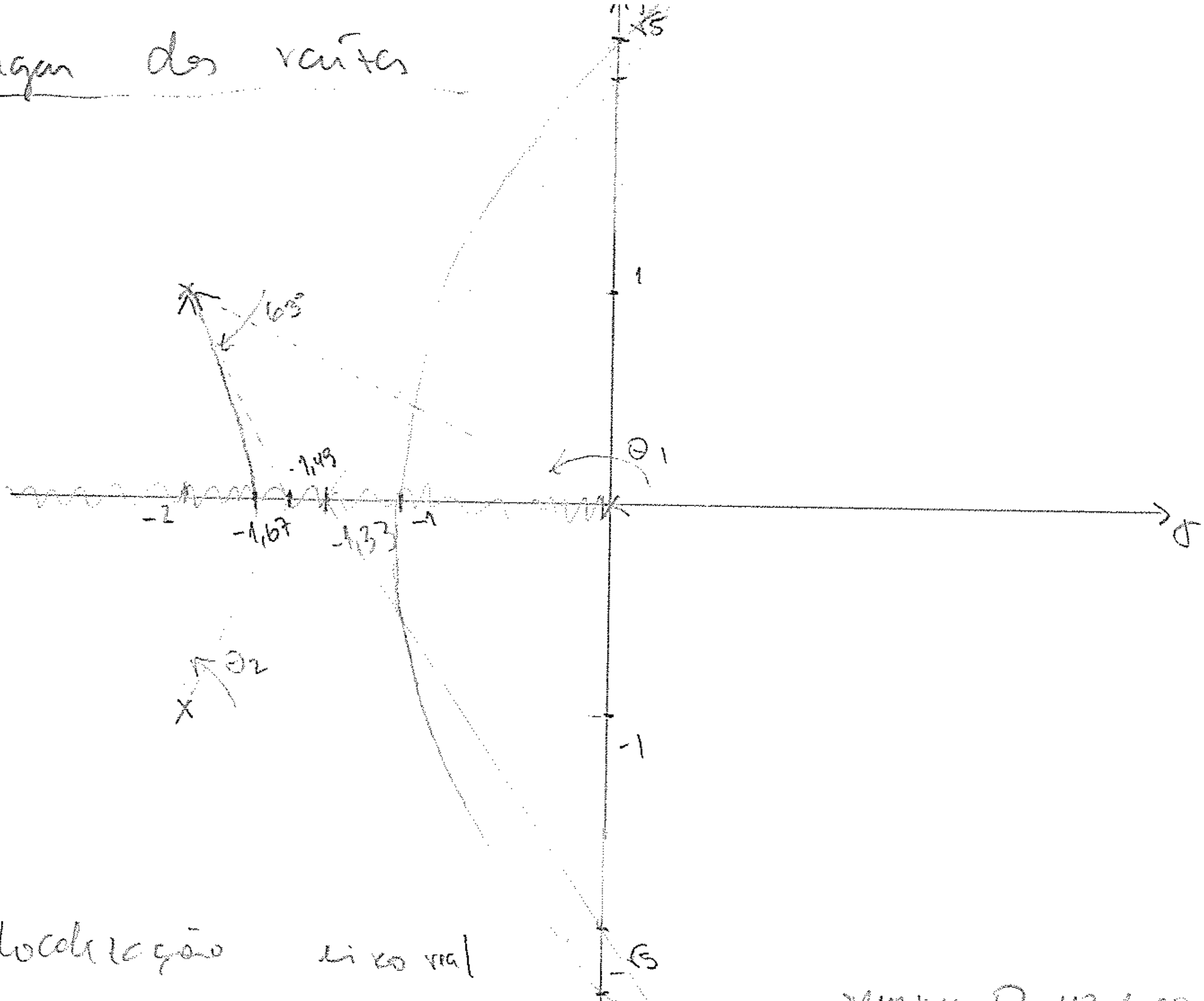
$$\left[(B_2 s + 4K_1)\frac{(M_1 s^2 + B_1 s + K_1)}{K_1} - 4K_1\right] X(s) = 4F(s)$$

$$(M_1 B_2 s^3 + B_1 B_2 s^2 + K_1 B_2 s + 4K_1 M_1 s^2 + 4K_1 B_1 s + 4K_1^2 - 4K_1^2) X(s) = 4K_1 F$$

$$\left[M_1 B_2 s^3 + (B_1 B_2 + 4K_1 M_1) s^2 + (K_1 B_2 + 4K_1 B_1) s\right] X(s) = 4K_1 F(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{8}{8s^3 + 32s^2 + 40s} = \frac{1}{s^3 + 4s^2 + 5s}$$

Lugar dos raízes




2) Localização eixo real

→ sempre 0 no eixo real

$\angle G(s)H(s) = -\angle s - \angle s^2+4s+5$

$p/\ \sigma>0 \Rightarrow -0 - 0 = 0$ ×

$p\ \sigma<0 \quad -180 - 0 = -180$ ✓

3) assíntotas:

$$\text{intersecção} = -\frac{\Sigma p_i - \Sigma z_i}{n-m} = -\frac{(4)-0}{3} = -1,33$$

4) $$1 + \frac{K}{s(s^2+4s+5)} = 0 \Rightarrow K = -(s^3+4s^2+5s)$$

$$\frac{dK}{ds} = 0 = 3s^2+8s+5 \Rightarrow s = \begin{cases} -1,67 \\ -1 \end{cases}$$

5) ângulo de saída do pólo

$$\alpha = 180° - \theta_1 - \theta_2 = 180 - 153° - 90° = -63{,}4°$$

$$\theta_1 = -\operatorname{atan}\left(\frac{1}{2}\right) + 180° = 153{,}4°$$

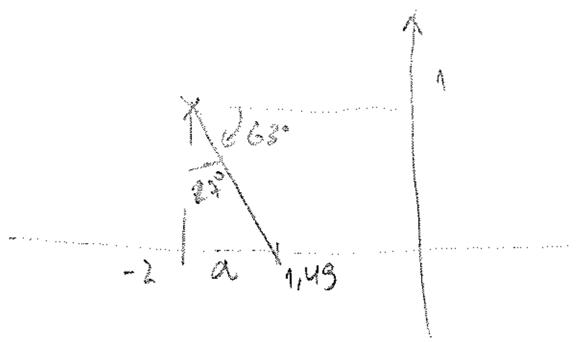
6)

$$s^3 + 4s^2 + 5s + K = 0 \quad ; \quad s = j\omega$$

$$-j\omega^3 - 4\omega^2 + 5j\omega + K = 0$$

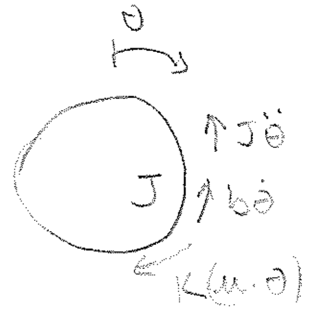
$$j\omega(5 - \omega^2) + K - 4\omega^2 = 0$$

$$\begin{cases} 5 - \omega^2 = 0 \\ K - 4\omega^2 = 0 \end{cases} \rightarrow \begin{cases} \omega = \pm\sqrt{5} = \pm 2{,}23 \\ K - 4 \cdot 5 = 0 \Rightarrow K = 20 \end{cases}$$




$$\tan 27 = \frac{a}{1} \rightarrow a = 0{,}51$$

5)




$-J\ddot{\theta} - b\dot{\theta} + Ku - K\theta = 0$

$KU(s) = Js^2\Theta(s) + bs\Theta(s) + K\Theta(s)$

$$\frac{\Theta(s)}{U(s)} = \frac{K}{Js^2 + bs + K} = \frac{K/J}{s^2 + \frac{b}{J}s + \frac{K}{J}}$$

$J = 1,\ K = 2,\ b = 0.4$

$\omega_n = \sqrt{\frac{K}{J}} = 2,$

$2\xi\omega_n = \frac{b}{J}$

$\xi = \frac{b}{2\omega_n J} = 0.1$

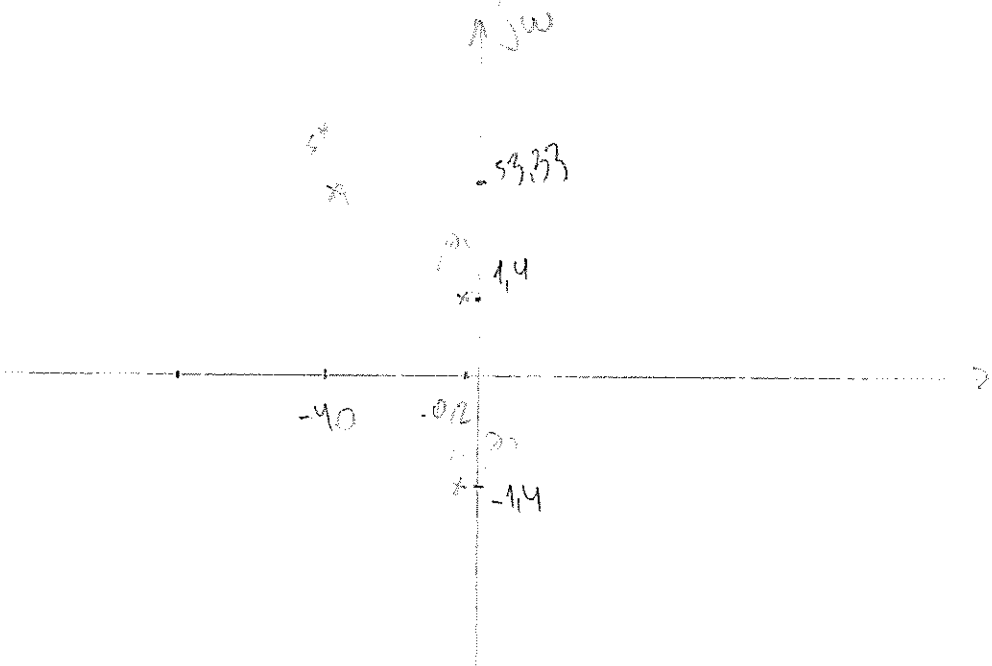
$t_s = \frac{4}{\xi\omega_n} = 20 \Rightarrow \omega_n = 2,$

$$\frac{\Theta(s)}{U(s)} = \frac{2}{s^2 + 0{,}4s + 2} \rightarrow s_{1,2} = -0{,}2 \pm j1{,}4$$

Requisitos: $t_{s_{2\%}} = 0{,}1s$ ; $\xi = 0{,}6 \Rightarrow \omega_n = 66{,}7$

$s^* = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

$= -40 \pm j53{,}33$



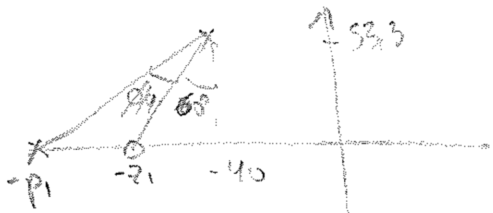
$$\angle G(s^*)H(s^*) = -\angle s^{*2} + 10{,}4s + 14 = -\theta_1 - \theta_2$$

$$= -\left[180 - \operatorname{atan}\left(\frac{53{,}3 - 1{,}4}{39{,}8}\right)\right] - \left[180 - \operatorname{atan}\left(\frac{53{,}3 + 1{,}4}{39{,}8}\right)\right]$$

$$= -253^\circ$$

Logo $\phi = 73^\circ$. Vamos projetar uma compensada por avanço de fase tal que

$$\phi/4 = 18{,}4^\circ \quad ; \quad G_c(s) = K_c\left(\frac{s + z_1}{s + p_1}\right)^4$$




$$60^\circ + \frac{\phi}{4} < 90^\circ \ \checkmark$$

$$\tan 60^\circ = \frac{(z_1 - 40)}{53{,}33} \Rightarrow z_1 = 132{,}4$$

$$\tan\left(\phi/4 + 60^\circ\right) = \frac{p_1 - 40}{53{,}33} \Rightarrow p_1 = 299{,}4$$

$$\left| K_c \, G_c(s) \, G(s) \right|_{s = s^*} = 1$$

$$K_c \cdot 0{,}0263 \cdot 9{,}0347 \cdot 10^{-4} = 1 \quad \therefore K_c = 4{,}204 \cdot 10^{4}$$