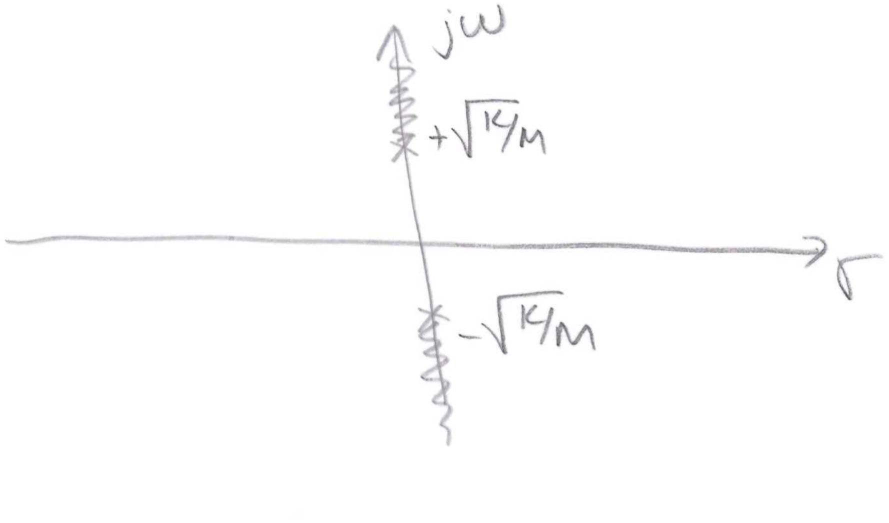
11/4 — PROVA I DE CONTROLE DE SISTEMAS

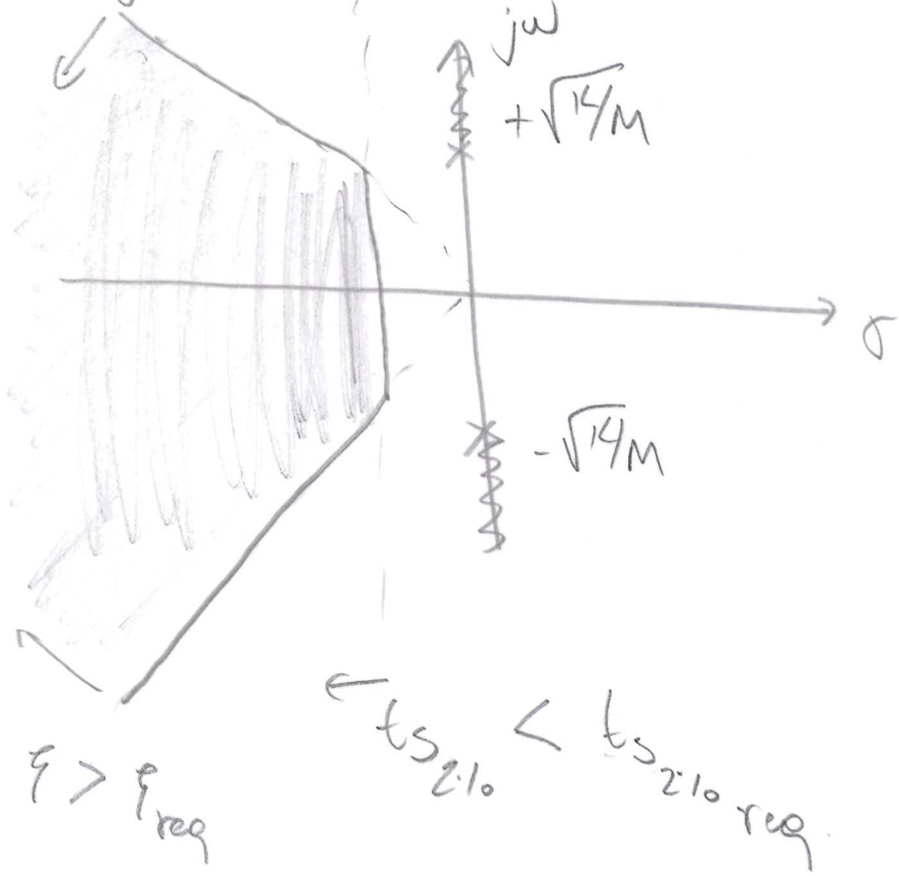
a) $$F(s) = Ms^2X(s) + KX(s)$$
$$= (Ms^2 + K)X(s) \Rightarrow \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + K}$$

b) $$\frac{X(s)}{F(s)} = \frac{1/M}{s^2 + K/M} = \frac{1}{K} \cdot \frac{K/M}{s^2 + K/M}$$
$$\Rightarrow \omega_n^2 = K/M \Rightarrow \omega_n = \sqrt{\frac{K}{M}}$$
$$2\xi\omega_n = 0 \Rightarrow \xi = 0$$

c) Pólos: $s^2 + K/M = 0 \Rightarrow s_{1,2} = \pm j\sqrt{K/M}$

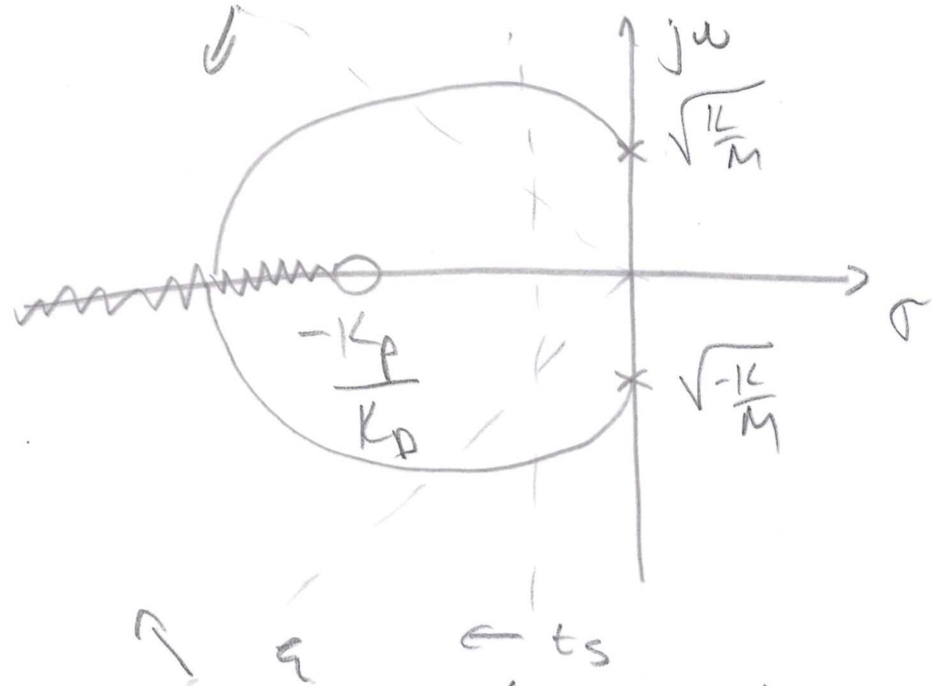


d) $t_{s_{2\%}} = \frac{4}{\sigma}$



A REGIÃO HACHURADA É A INTERSEÇÃO ENTRE OS REQUISITOS t_s E ξ (AMBOS SÃO SATISFEITOS)

É NECESSÁRIO CORRIGIR A FASE DO SIST. EM MALHA FECHADA DE MODO QUE OS RAMOS DO L.R. TENHAM INCURSÕES NESSA REGIÃO FACTÍVEL e)



$$G_c(s)\,G(s)\,H(s) = (K_p + K_D s)\,\frac{1}{Ms^2 + K}\cdot 1$$

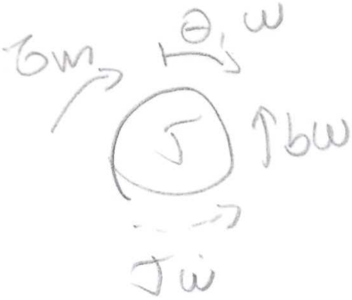
$$= \frac{K_p + K_D s}{Ms^2 + K}$$

Zeros : $-K_p/K_D$

Pólos : $\pm j\sqrt{\frac{K}{M}}$

$\Rightarrow$ sabendo que o LR começa nos pólos de MA e termina nos zeros de MA ou nas assíntotas, é possível alocar o zero em $-K_p/K_D$ de modo que haja partes do L.R. dentro da região factível de projeto

2)



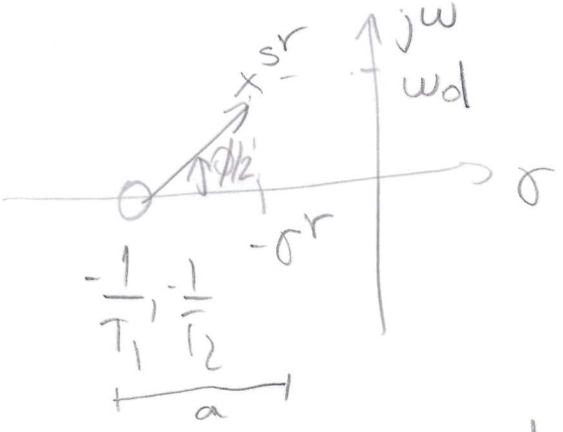
$$\sum \tau_i = 0$$

$$\tau_m - b\omega - J\dot{\omega} = 0$$

$$K_\tau E_i - bs\theta - Js^2\theta = 0$$

$$\frac{\theta(s)}{E_i(s)} = \frac{K_\tau}{Js^2 + bs} = \frac{K_\tau}{s(Js+b)}$$

Requisitos: $t_s = \frac{4}{\sigma} \rightarrow \omega_n = \frac{4}{t_s \cdot \xi}$



$$\tan \phi/2 = \frac{\omega d}{a} \rightarrow a = \frac{\omega d}{\tan \phi/2}$$

$$\frac{1}{T_1} = a - \sigma^r \Rightarrow T_1 = T_2 = \frac{1}{a - \sigma^r}$$

COND. MODULO:

$$|G_c^{PID} \cdot G|_{s=s^r} = 1$$

$$K \cdot |G_{c1} \cdot G|_{s=s^r} = 1$$

$$K = \frac{1}{|G_{c1} \cdot G|_{s=s^r}}$$

$$G_c^{PID} = K_P + \frac{K_I}{s} + K_D s$$

$$= \frac{K_D s^2 + K_P s + K_I}{s}$$

```
J = 3.7e-3; b = 4.6e-2; ktau = 4.33e-2;

s = tf('s')
```

```
s =

  s

Continuous-time transfer function.
```

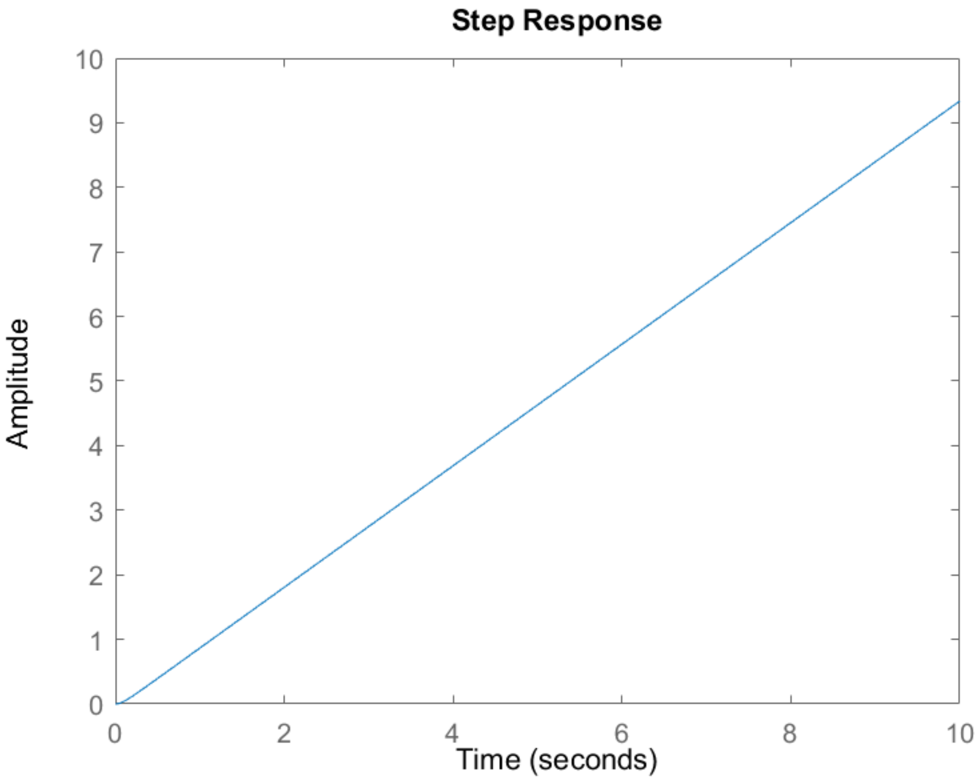
```
G = ktau/(s*(J*s+b))
```

```
G =

        0.0433
  --------------------
  0.0037 s^2 + 0.046 s

Continuous-time transfer function.
```

```
damp(G)
```

```
    Pole         Damping       Frequency      Time Constant
                              (rad/seconds)     (seconds)

   0.00e+00     -1.00e+00       0.00e+00           Inf
  -1.24e+01      1.00e+00       1.24e+01        8.04e-02
```

```
figure, step(G,10)
```




```
ts = 1
```

```
ts = 1
```

```
zeta = 0.7
```

```
zeta = 0.7000
```

```
omn = 4/(ts*zeta)
```

```
omn = 5.7143
```

```
sr = -zeta*omn + 1i*omn*sqrt(1-zeta^2)
```

```
sr = -4.0000 + 4.0808i
```

```
sigmar = real(sr)
```

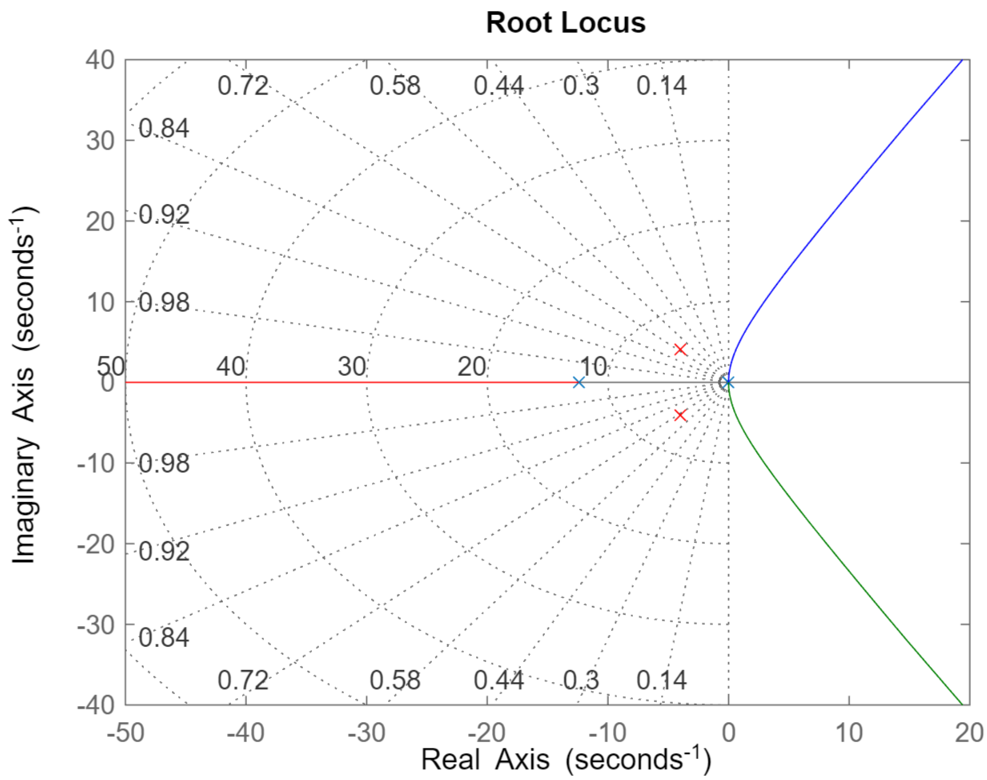
```
sigmar = -4
```

```
omdr = imag(sr)
```

```
omdr = 4.0808
```

```
figure
rlocus(G/s), hold on, grid on
plot(sr,'rx')
```

```
plot(conj(sr),'rx')
```




```
% xlim([-3 1])

phG = rad2deg(angle(evalfr(G/s,sr)))
```

```
phG = 65.3217
```

```
phi = 180-phG
```

```
phi = 114.6783
```

```
phi2 = phi/2
```

```
phi2 = 57.3392
```

```
a= omdr/tand(phi2)
```

```
a = 2.6159
```

```
T1 = 1/(a-sigmar)
```

```
T1 = 0.1512
```

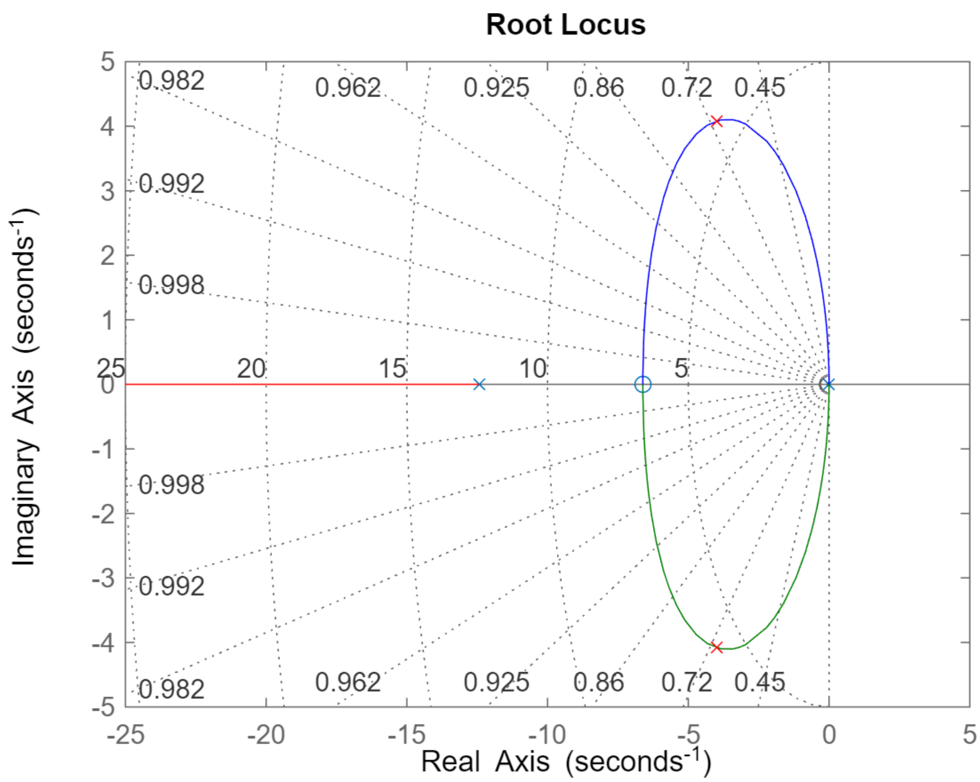
```
T2 = T1
```

```
T2 = 0.1512
```

```
Gc1 = (T1*s+1)*(T2*s+1)/s % parte dinamica do compensador PID
```

```
Gc1 =

  0.02285 s^2 + 0.3023 s + 1
  --------------------------
              s

Continuous-time transfer function.
```

```
figure
rlocus(G*Gc1), hold on, grid on
plot(sr,'rx')
plot(conj(sr),'rx')
```




```
% xlim([-3 1])

% cond modulo
K = 1/abs(evalfr(G*Gc1,sr))
```

```
K = 48.6931
```

```
GcPID = K*Gc1
```

```
GcPID =

  1.112 s^2 + 14.72 s + 48.69
  ---------------------------
               s

Continuous-time transfer function.
```

```
Kd= GcPID.Numerator{1}(1)
```

```
Kd = 1.1125
```

```
Kp= GcPID.Numerator{1}(2)
```

```
Kp = 14.7200
```

```
Ki= GcPID.Numerator{1}(3)
```

```
Ki = 48.6931
```

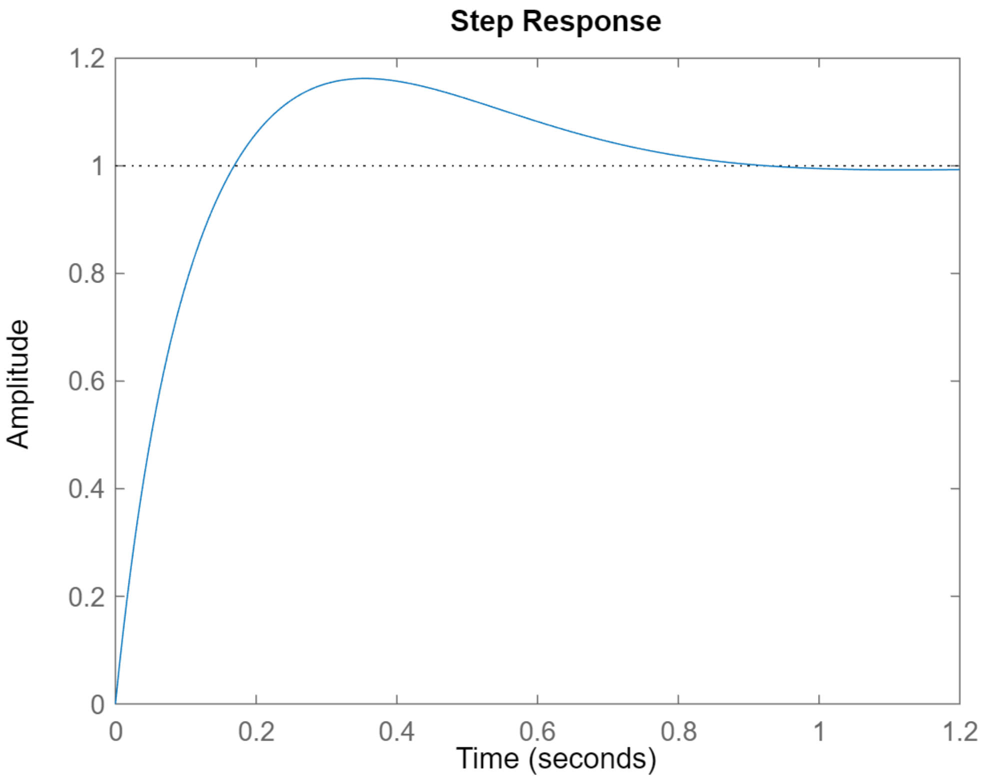
```
Gmf = zpk(feedback(GcPID*G,1))
```

```
Gmf =
 
       13.019 (s+6.616)^2
  ----------------------------
  (s+17.45) (s^2 + 8s + 32.65)
 
Continuous-time zero/pole/gain model.
```

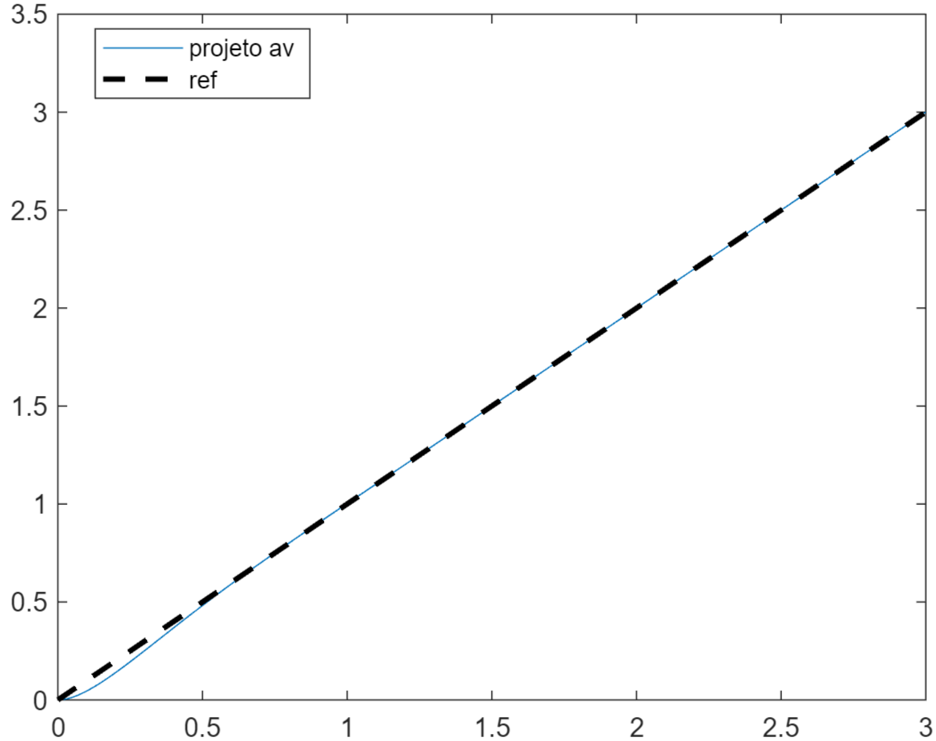
```
damp(Gmf)
```

```
         Pole              Damping       Frequency      Time Constant  
                                       (rad/seconds)      (seconds)    
                                                                       
 -4.00e+00 + 4.08e+00i     7.00e-01       5.71e+00         2.50e-01    
 -4.00e+00 - 4.08e+00i     7.00e-01       5.71e+00         2.50e-01    
 -1.75e+01                 1.00e+00       1.75e+01         5.73e-02    
```

```
figure, step(Gmf)
```




```
[y,t] = step(Gmf/s,3);
figure,
plot(t,y), hold on %45 degree line
plot(t,t,'k--','LineWidth',2)
legend('projeto av','ref','location','best')
```


projeto av
ref
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3