Prova 1 - MSD 2019.1

1) Prove que a equivalência da representação em espaço de estados

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

é a função de transferência

$$G(s) = C(sI-A)^{-1}B + D$$

A definição da FT é

$$G(s) = \left.\frac{Y(s)}{U(s)}\right|_{\text{CI's nulas}}$$

Logo temos

$$\begin{cases} sX(s) = AX(s) + BU(s) \\ Y(s) = CX(s) + DU(s) \end{cases}$$

$$sX(s) - AX(s) = BU(s)$$

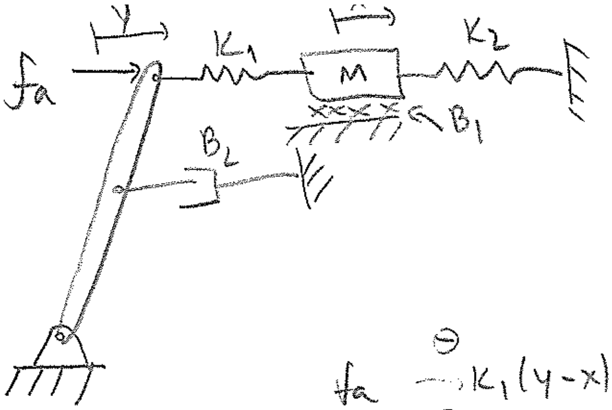
$$(sI - A)X(s) = BU(s)$$

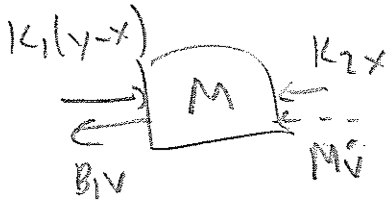
$$X(s) = (sI-A)^{-1}BU(s)$$

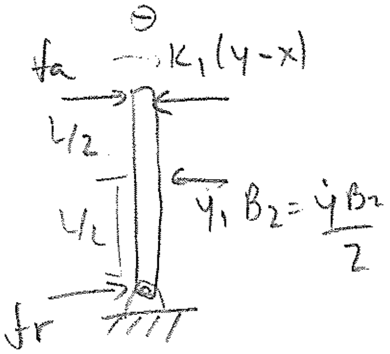
$$Y(s) = C(sI-A)^{-1}BU(s) + DU(s)$$

$$\frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D$$

2)







$$\tan\theta = \frac{y_1}{L/2} = \frac{y}{L}$$

$$y_1 = \frac{y}{2}$$

$$\dot{y}_1 = \dot{y}/2$$

$$L f_a - L K_1(y-x) - \frac{L}{2}\frac{\dot{y}B_2}{2} = 0$$

$$L f_a - L K_1 y + L K_1 x - \frac{L B_2}{4}\dot{y} = 0$$

(I) $$\dot{y} = \frac{4K_1}{B_2}x - \frac{4K_1}{B_2}y + \frac{4}{B_2}f_a$$

$$-M\dot{v} - K_2 x - B_1 v + K_1 y - K_1 x = 0$$

(II) $$\dot{v} = -\frac{K_1+K_2}{M}x - \frac{B_1}{M}v + \frac{K_1}{M}y$$

(III) $$\dot{x} = v$$

$$\dot{x} = Ax + Bu$$

$$x = \begin{bmatrix} y \\ v \\ x \end{bmatrix}; \quad A = \begin{bmatrix} -\frac{4K_1}{B_2} & 0 & \frac{4K_1}{B_2} \\ \frac{K_1}{M} & -\frac{B_1}{M} & -\frac{(K_1+K_2)}{M} \\ 0 & 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 4/B_2 \\ 0 \\ 0 \end{bmatrix}$$

$u = f_a$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

b) FT

① $B_2 s Y(s) = 4K_1 X(s) - 4K_1 Y(s) + 4 F_a(s)$

② $Ms^2 X(s) = -(K_1+K_2) X(s) - B_1 s X(s) + K_1 Y(s)$

$X(s)\left[Ms^2 + (K_1+K_2) + B_1 s\right] = K_1 Y(s)$

$X(s) = \dfrac{K_1}{Ms^2 + B_1 s + K_1 + K_2} Y(s)$ ✓

② → ① $Y(s)\left[B_2 s + 4K_1\right] - \dfrac{4K_1^2}{Ms^2 + B_1 s + K_1 + K_2} Y(s) = 4F_a(s)$

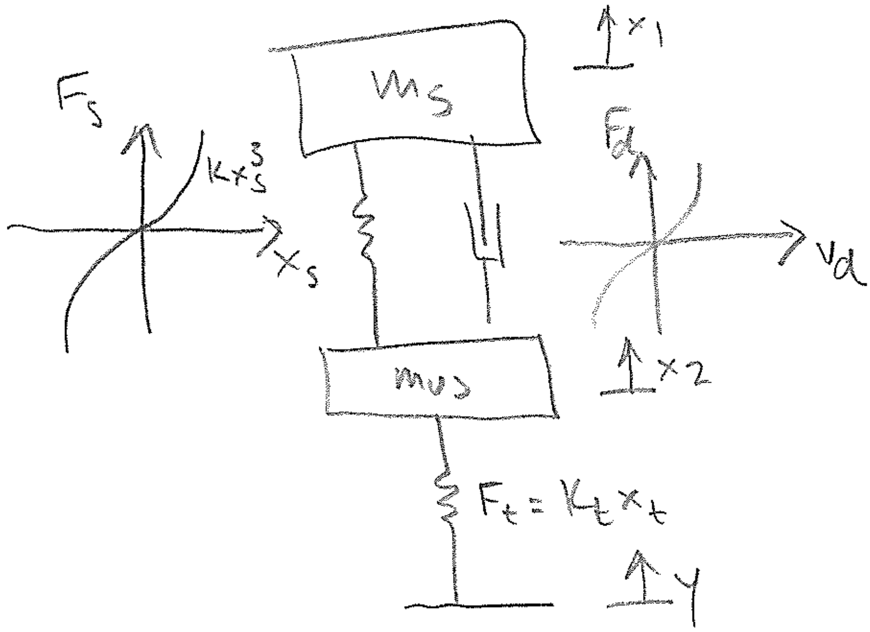
$Y(s)\begin{bmatrix} B_2 M s^3 + B_1 B_2 s^2 + B_2(K_1+K_2) s + \\ 4K_1 M s^2 + 4K_1 B_1 s + 4K_1(K_1+K_2) - 4K_1^2 \end{bmatrix} = 4(Ms^2 + B_1 s + K_1 + K_1) F_a$

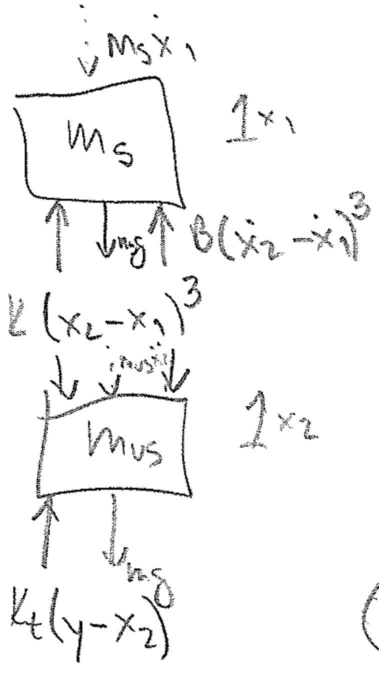
$$\frac{Y(s)}{F_a(s)} = \frac{4(Ms^2 + B_1 s + K_1 + K_2)}{B_2 M s^3 + (B_1 B_2 + 4K_1 M)s^2 + \left[B_2(K_1+K_2) + 4K_1 B_1\right]s + \cancel{4K_1^2} + 4K_1 K_2 - 4K_1^2}$$

$$= \frac{4Ms^2 + 4B_1 s + 4(K_1+K_2)}{B_2 M s^3 + (B_1 B_2 + 4K_1 M)s^2 + \left[B_2(K_1+K_2) + 4K_1 B_1\right]s + 4K_1 K_2}$$ ✓

$$= \frac{\overbrace{\frac{4}{B_2}}^{b_0} s^2 + \overbrace{\frac{4B_1}{B_2 M}}^{b_1} s + \overbrace{\frac{4(K_1+K_2)}{B_2 M}}^{b_2}}{s^3 + \underbrace{\frac{B_1 B_2 + 4K_1 M}{B_2 M}}_{a_1} s^2 + \underbrace{\frac{\left[B_2(K_1+K_2) + 4K_1 B_1\right]}{B_2 M}}_{a_2} s + \underbrace{\frac{4K_1 K_2}{B_2 M}}_{a_3}}$$

3)





$$-m_s\ddot{x}_1 - m_s g + K(x_2-x_1)^3 + B(\dot{x}_2-\dot{x}_1)^3 = 0$$

(1) $$\ddot{x}_1 = \frac{K}{m_s}(x_2-x_1)^3 + \frac{B}{m_s}(\dot{x}_2-\dot{x}_1)^3 - g$$

$$-m_{us}\ddot{x}_2 - K(x_2-x_1)^3 - B(\dot{x}_2-\dot{x}_1)^3 - mg + K_t(y- \quad = 0$$

(2) $$\ddot{x}_2 = -\frac{K}{m_{us}}(x_2-x_1)^3 - \frac{B}{m_{us}}(\dot{x}_2-\dot{x}_1)^3 - g + \frac{K_t}{m_{us}}(y-$$

D) o vetor de estado:

$$\underline{z} = [x_1 \ \dot{x}_1 \ x_2 \ \dot{x}_2]^T, \quad u = [y];$$

$$\dot{\underline{z}} = \begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ 1 \\ \dot{x}_2 \\ 2 \end{bmatrix} = \underline{f}[\underline{z}, u]$$