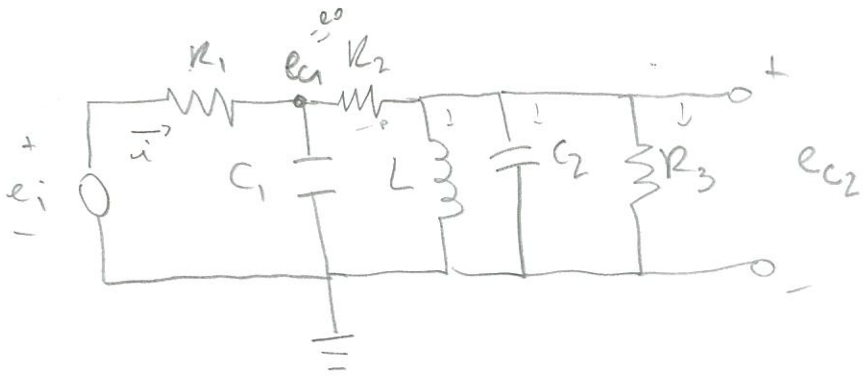
Prova 2 MSD 2013.1

1) $x = \begin{bmatrix} e_{c1} \\ e_{c2} \\ i_L \end{bmatrix}$; $u = e_i$, $y = i$



(I) $\dot{e}_{c1} = \frac{1}{C_1} i_{c1}$

$i = i_{c1} + i_{R2}$

$\frac{e_i - e_0}{R_1} = i_{c1} + \frac{e_0 - e_{c2}}{R_2}$

$i_{c1} = \frac{R_2 e_i - R_2 e_0 - R_1 e_0 + R_1 e_{c2}}{R_1 R_2}$

$i_{c1} = \frac{1}{R_1} e_i + \frac{1}{R_2} e_{c2} - \frac{(R_1+R_2)}{R_1 R_2} e_0$

$\dot{e}_{c1} = \frac{1}{C_1 R_1} e_i + \frac{1}{C_1 R_2} e_{c2} - \frac{(R_1+R_2)}{C_1 R_1 R_2} e_{c1}$

(II) $\dot{e}_{c2} = \frac{1}{C_2} i_{c2}$

$i_{R2} = i_L + i_{c2} + i_{R3}$

$\frac{e_{c1} - e_{c2}}{R_2} = i_L + i_{c2} + \frac{e_{c2}}{R_3}$

$i_{c2} = \frac{1}{R_2} e_{c1} - \frac{1}{R_2} e_{c2} - i_L - \frac{1}{R_3} e_{c2}$

$i_{c2} = \frac{1}{R_2} e_{c1} - \frac{R_3+R_2}{R_2 R_3} e_{c2} - i_L$

$\dot{e}_{c2} = \frac{1}{C_2 R_2} e_{c1} - \frac{R_2+R_3}{C_2 R_2 R_3} e_{c2} - \frac{1}{C_2} i_L$; $\frac{e_i - e_{c1}}{R_1} = i$

(III) $\frac{di_L}{dt} = \frac{1}{L} e_L = \frac{1}{L} e_{c2}$

$i = \frac{1}{R_1} e_i - \frac{1}{R_1} e_{c1}$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

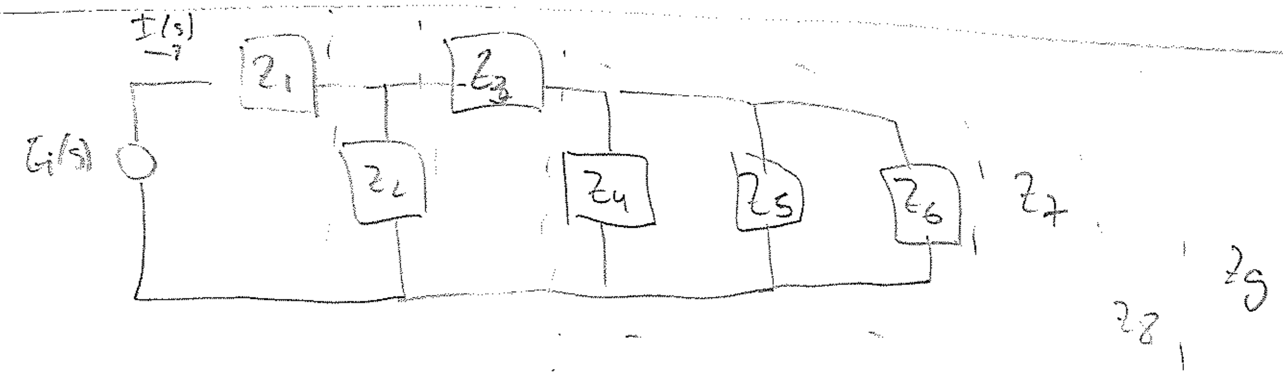
a)

$$\begin{bmatrix} \dot{e}_{c1} \\ \dot{e}_{c2} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} e_{c1} & e_{c2} & i_L \end{array} \\ \begin{bmatrix} -\frac{(R_1+R_2)}{C_1R_1R_2} & 1/C_1R_2 & 0 \\ 1/C_2R_2 & -\frac{(R_2+R_3)}{C_2R_2R_3} & -1/C_2 \\ 0 & 1/L & 0 \end{bmatrix} \end{array} \begin{bmatrix} e_{c1} \\ e_{c2} \\ i_L \end{bmatrix} + \begin{bmatrix} 1/C_1R_1 \\ 0 \\ 0 \end{bmatrix} e$$

$$\begin{bmatrix} i \end{bmatrix} = \begin{bmatrix} -1/R_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{c1} \\ e_{c2} \\ i_L \end{bmatrix} + \begin{bmatrix} 1/R_1 \end{bmatrix} e$$

//

b)



$$\frac{1}{Z_7} = \frac{1}{Z_4} + \frac{1}{Z_5} + \frac{1}{Z_6}$$

$$Z_8 = Z_3 + Z_7$$

$$\frac{1}{Z_9} = \frac{1}{Z_2} + \frac{1}{Z_8}$$

$$Z_{eq} = Z_1 + Z_9$$

$$\begin{cases} Z_1 = R_1 = 2 \; ✓ \\ Z_2 = \frac{1}{sC_1} = \frac{1}{s} \; ✓ \\ Z_3 = R_2 = 2 \; ✓ \\ Z_4 = Ls = s/2 \; ✓ \\ Z_5 = \frac{1}{sC_2} = \frac{1}{4s} \; ✓ \\ Z_6 = R_3 = 4 \; ✓ \end{cases}$$

$$\frac{1}{Z_7} = \frac{2}{s} + 4s + \frac{1}{4} = \frac{8 + 16s^2 + s}{4s}$$

$$Z_7 = \frac{4s}{16s^2 + s + 8}$$

$$Z_8 = 2 + \frac{4s}{16s^2 + s + 8} = \frac{32s^2 + 2s + 16 + 4s}{16s^2 + s + 8}$$

$$= \frac{32s^2 + 6s + 16}{16s^2 + s + 8}$$

$$\frac{1}{Z_9} = s + \frac{16s^2 + s + 8}{32s^2 + 6s + 16} = \frac{32s^3 + 6s^2 + 16s + 16s^2 + s + 8}{32s^2 + 6s + 16}$$

$$= \frac{32s^3 + 22s^2 + 17s + 8}{32s^2 + 6s + 16}$$

$$Z_9 = \frac{32s^2 + 6s + 16}{32s^3 + 22s^2 + 17s + 8}$$

$$Z_{eq} = \frac{32s^2 + 6s + 16 + 64s^3 + 44s^2 + 34s + 16}{32s^3 + 22s^2 + 17s + 8}$$

$$= \frac{64s^3 + 76s^2 + 40s + 32}{32s^3 + 22s^2 + 17s + 8} \quad //$$

$$E_i(s) = I(s)\, Z_{eq}$$

$$\frac{I(s)}{E_i(s)} = \frac{1}{Z_{eq}} = \frac{32s^3 + 22s^2 + 17s + 8}{64s^3 + 76s^2 + 40s + 32} \quad //$$

a) $A = \begin{bmatrix} -1 & 1/2 & 0 \\ 1/8 & -3/16 & -1/4 \\ 0 & 2 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 1/2 \\ 0 \\ 0 \end{bmatrix}$; $C = \begin{bmatrix} -1/2 & 0 & 0 \end{bmatrix}$

(valores numéricos)

$D = [1/2]$ //

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2) a)

$$x = [\, p_1 \quad p_2 \quad x_s \quad \dot{x}_s \quad x_t \quad \dot{x}_t \,]^T$$

$$u = x_c$$

b)

$$Q_1 = C_d\, A_1 \sqrt{\frac{2}{\rho} \Delta P_1} \quad ; \quad \Delta P_1 = \begin{cases} P_s - P_1 \,, & x_c > 0 \\ P_1 - P_r \,, & x_c < 0 \\ 0 \,, & x_c = 0 \end{cases}$$

$$= C_d\, x_c h \sqrt{\frac{2}{\rho} \Delta P_1}$$

$$A_1 = x_c h$$

$$Q_2 = C_d\, A_2 \sqrt{\frac{2}{\rho} \Delta P_2} \quad ; \quad \Delta P_2 = \begin{cases} P_2 - P_r \,, & x_c > 0 \\ P_s - P_2 \,, & x_c < 0 \\ 0 \,, & x_c = 0 \end{cases}$$

$$= -C_d\, x_c h \sqrt{\frac{2}{\rho} \Delta P_2}$$

$$A_2 = -x_c h$$

$$\dot{P}_1 = \frac{\beta}{V_1} (Q_1 - \dot{V}_1) \quad ; \quad V_1 = V_0 + (T - x_s) A_1$$

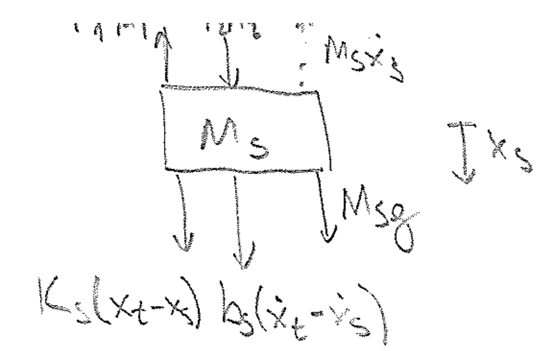
$$\dot{V}_1 = -\dot{x}_s A_1$$

(I) $$= \frac{\beta}{V_0 + (T - x_s) A_1} (Q_1 + \dot{x}_s A_1)$$

$$\dot{P}_2 = \frac{\beta}{V_2} (Q_2 - \dot{V}_2) \quad ; \quad V_2 = V_0 + A_2 x_s$$

$$\dot{V}_2 = \dot{x}_s A_2$$

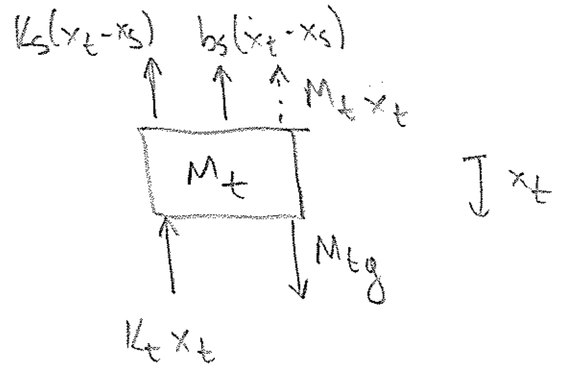
(II) $$= \frac{\beta}{V_0 + A_2 x_s} (Q_2 - A_2 \dot{x}_s)$$



$$\sum F_i = 0$$

$$-M_s\ddot{x}_s + P_2A_2 - P_1A_1 + M_s g + b_s(\dot{x}_t-\dot{x}_s) + $$
$$+ K_s(x_t-x_s) = 0$$

$$\ddot{x}_s = -\frac{K_s}{M_s}x_s + \frac{K_s}{M_s}x_t - \frac{b_s}{M_s}\dot{x}_s + \frac{b_s}{M_s}\dot{x}_t + $$
$$-\frac{A_1}{M_s}P_1 + \frac{A_2}{M_s}P_2 + g \qquad (\mathrm{III})$$



$$\sum F_i = 0$$

$$-M_t\ddot{x}_t - b_s(\dot{x}_t-\dot{x}_s) - K_s(x_t-x_s) + $$
$$+ M_t g - K_t x_t = 0$$

$$\ddot{x}_t = \frac{K_s x_s}{M_t} - \frac{(K_t+K_s)}{M_t}x_t + \frac{b_s}{M_t}\dot{x}_s - \frac{b_s}{M_t}\dot{x}_t + g \qquad \boxed{\mathrm{IV}}$$

$$f_1(x,u) = \dot{P}_1 \qquad (\mathrm{I})$$
$$f_2(x,u) = \dot{P}_2 \qquad (\mathrm{II})$$
$$f_3(x,u) = \dot{x}_s$$
$$f_4(x,u) = \ddot{x}_s \qquad (\mathrm{III})$$
$$f_5(x,u) = \dot{x}_t$$
$$f_6(x,u) = \ddot{x}_t \qquad (\mathrm{IV})$$

3) a) $x = [\; i \quad \theta_1 \quad \omega_1 \quad \theta_2 \quad \omega_2 \;]$

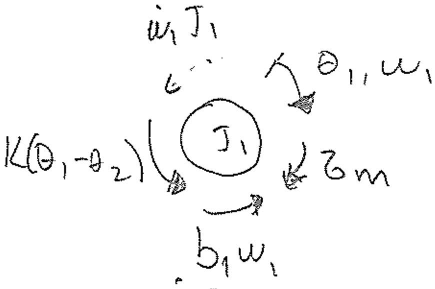
$u = [\; e_i \quad \tau_f \;]^T$

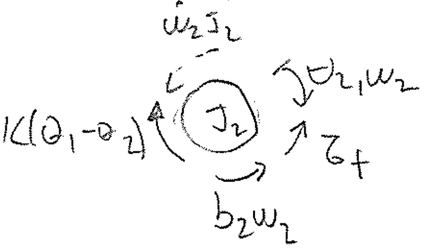
b) Circuito:

$$e_R + e_L + e_{cf} - e_1 = 0$$

$$Ri + L\frac{di}{dt} + K_c\omega_1 - e_i = 0$$

$$\frac{di}{dt} = -\frac{R}{L} i - \frac{K_c}{L}\omega_1 + \frac{1}{L} e_i$$





$$\sum \tau_i = 0$$

$$K_m i - b_1\omega_1 - K(\theta_1-\theta_2) - \dot{\omega}_1 J_1 = 0$$

$$\dot{\omega}_1 = \frac{K_m}{J_1} i - \frac{K}{J_1}\theta_1 - \frac{b_1}{J_1}\omega_1 + \frac{K}{J_1}\theta_2$$

$$-\tau_f - b_2\omega_2 + K(\theta_1-\theta_2) - \dot{\omega}_2 J_2 = 0$$

$$\dot{\omega}_2 = \frac{K}{J_2}\theta_1 - \frac{K}{J_2}\theta_2 - \frac{b_2}{J_2}\omega_2 - \frac{1}{J_2}\tau_f$$

$$\dot{x} = Ax + Bu \;;\; y = Cx + Du$$

$$\begin{bmatrix} di/dt \\ \dot{\theta}_1 \\ \dot{\omega}_1 \\ \dot{\theta}_2 \\ \dot{\omega}_2 \end{bmatrix} = \begin{bmatrix} -R/L & 0 & -K_c/L & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ K_m/J_1 & -K/J_1 & -b_1/J_1 & K/J_1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & K/J_2 & 0 & -K/J_2 & -b_2/J_2 \end{bmatrix} \begin{bmatrix} i \\ \theta_1 \\ \omega_1 \\ \theta_2 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} 1/L & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1/J_2 \end{bmatrix} \begin{bmatrix} e_i \\ \tau_f \end{bmatrix}$$

$$\begin{bmatrix} \tau_m \\ \omega_2 \end{bmatrix} = \begin{bmatrix} K_m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ \theta_1 \\ \omega_1 \\ \theta_2 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_i \\ \tau_f \end{bmatrix}$$