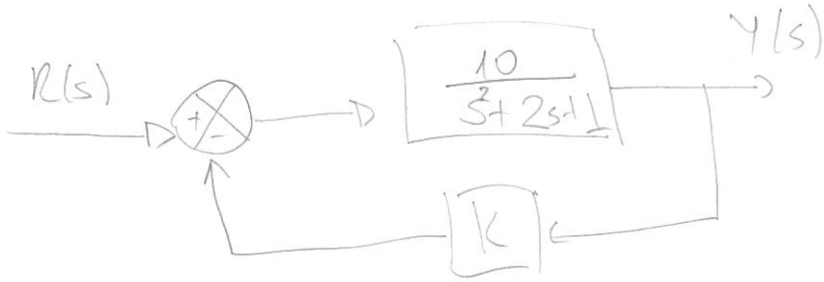
# PROVA 1 - MSD 2019.2

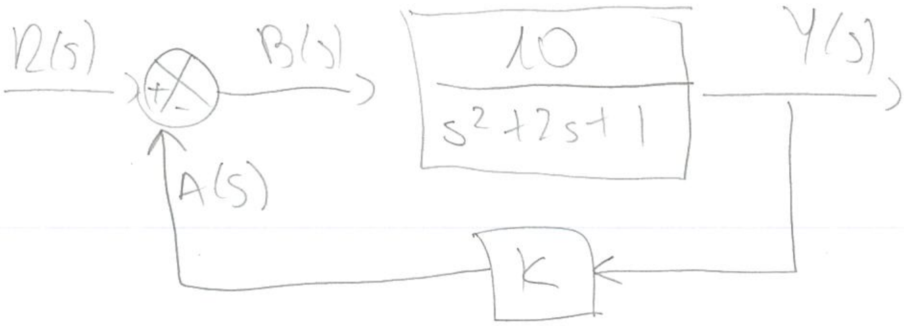
1)




a) Obter FT $\frac{Y(s)}{R(s)}$

b) Definir qual valor de K implica pólos do sistema imaginários-puros.

A) Definimos sinais auxiliares $A(s)$, $B(s)$:




QUE LEVA ÀS SEGUINTES RELAÇÕES:

$$\begin{cases} \dfrac{Y(s)}{B(s)} = \dfrac{10}{s^2+2s+1} = G(s) \\ A(s) = K\,Y(s) \\ B(s) = R(s) - A(s) \end{cases}$$

PARA FT $\frac{Y(s)}{R(s)}$ MANIPULAMOS ESTAS EQUAÇÕES

$\hookrightarrow B(s) = R(s) - K\,Y(s)$

$\hookrightarrow Y(s) = G(s)\left(R(s) - K\,Y(s)\right)$

$$Y(s)\left[1 + K\,G(s)\right] = G(s)\,R(s)$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + K\,G(s)}$$

$$= \frac{\dfrac{10}{s^2+2s+1}}{1 + \dfrac{10K}{s^2+2s+1}}$$

$$= \frac{10}{s^2+2s+1+10K} \quad //$$

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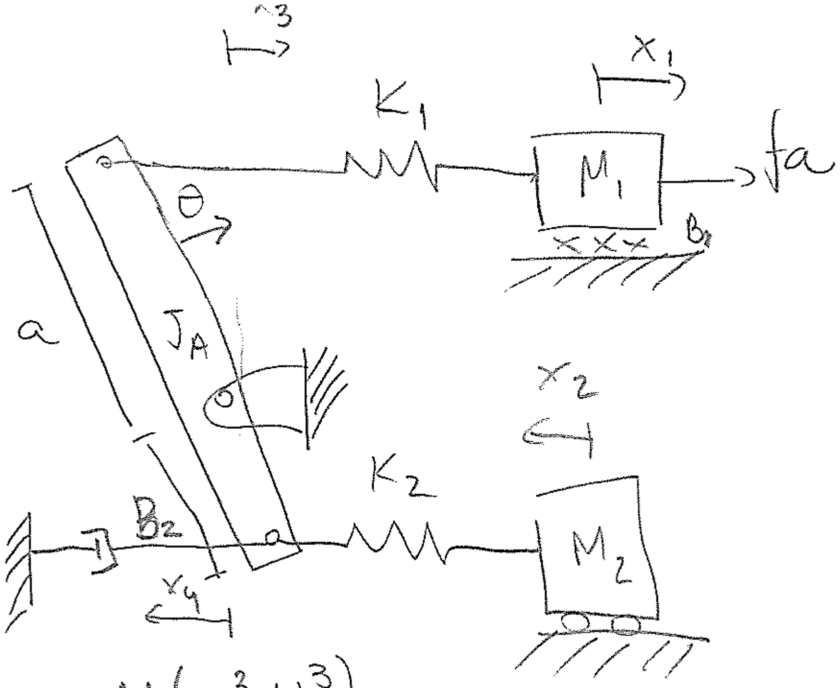
B) OS PÓLOS DO SISTEMA SÃO AS RAÍZES DA EQ. CARACTERÍSTICA

$$s^2 + 2s + 1 + 10K = 0$$

PÓLOS $\Rightarrow s_{1,2} = -1 \pm j\sqrt{10K}$

NÃO EXISTE $K$, TAL QUE OS PÓLOS SEJAM IMAGINÁRIOS-PUROS.

2)





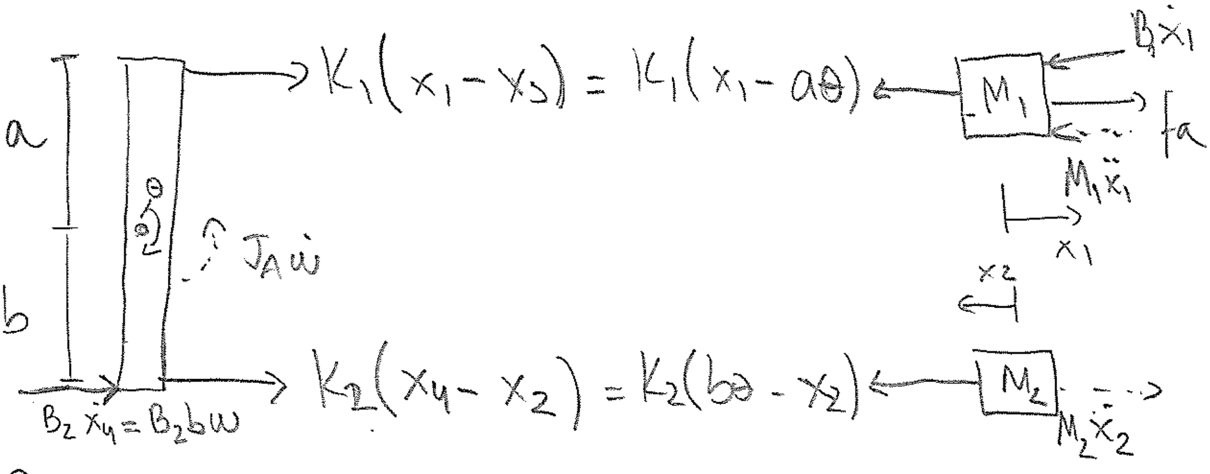

$$J_A = \frac{M(a^3+b^3)}{3(a+b)}$$ , PEQUENOS DESLOCAMENTOS PARA $\theta$

A) Espaço de Estados, para $y = \begin{bmatrix} \theta \\ \ddot{x}_1 \end{bmatrix}$

$$\operatorname{sen}\theta \cong \theta = \frac{x_3}{a} = \frac{x_4}{b}$$

$$\hookrightarrow \begin{cases} x_3 = a\theta \\ x_4 = b\theta \end{cases} \quad \therefore \quad \dot{x}_4 = b\omega$$

DCLs:




$$\sum_i \tau_i = 0$$

$$J_A:\ aK_1(x_1 - a\theta) - J_A\dot{\omega} - bK_2(b\theta - x_2) - b(B_2 b\omega) = 0$$

$$\dot{\omega} = \frac{aK_1}{J_A}x_1 - \frac{(a^2K_1 + b^2K_2)}{J_A}\theta + \frac{bK_2}{J_A}x_2 - \frac{b^2B_2}{J_A}\omega$$ //

$\sum_i F_i = 0$ ④

$M_1$: $\quad -K_1(x_1 - a\theta) - B_1\dot{x}_1 - M_1\ddot{x}_1 + fa = 0$

$$\ddot{x}_1 = \frac{-K_1}{M_1}x_1 - \frac{B_1}{M_1}\dot{x}_1 + \frac{K_1 a}{M_1}\theta + \frac{1}{M_1}fa$$

$M_2$: $\quad K_2(b\theta - x_2) - M_2\ddot{x}_2 = 0$

$$\ddot{x}_2 = \frac{-K_2}{M_2}x_2 + \frac{K_2 b}{M_2}\theta$$

Estados

$$\underline{z} = \begin{bmatrix} x_1 \\ x_2 \\ \theta \\ \dot{x}_1 \\ \dot{x}_2 \\ \omega \end{bmatrix}$$

ENTRADAS.

$$\underline{u} = [fa(t)]$$

SAÍDAS.

$$\underline{y} = \begin{bmatrix} \theta \\ \ddot{x}_1 \end{bmatrix}$$

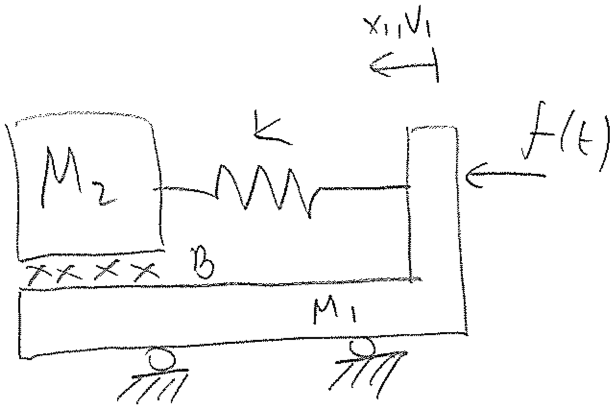
$$\dot{\underline{z}} = A\underline{z} + B\underline{u}, \quad \underline{y} = C\underline{z} + D\underline{u}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-K_1}{M_1} & 0 & \frac{K_1 a}{M_1} & -\frac{B_1}{M_1} & 0 & 0 \\ 0 & \frac{-K_2}{M_2} & \frac{K_2 b}{M_2} & 0 & 0 & 0 \\ \frac{aK_1}{J_A} & \frac{bK_2}{J_A} & -\frac{(a^2K_1 + b^2K_2)}{J_A} & 0 & 0 & \frac{-b^2B_2}{J_A} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \theta \\ \dot{x}_1 \\ \dot{x}_2 \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M_1 \\ 0 \\ 0 \end{bmatrix} [fa(t)]$$

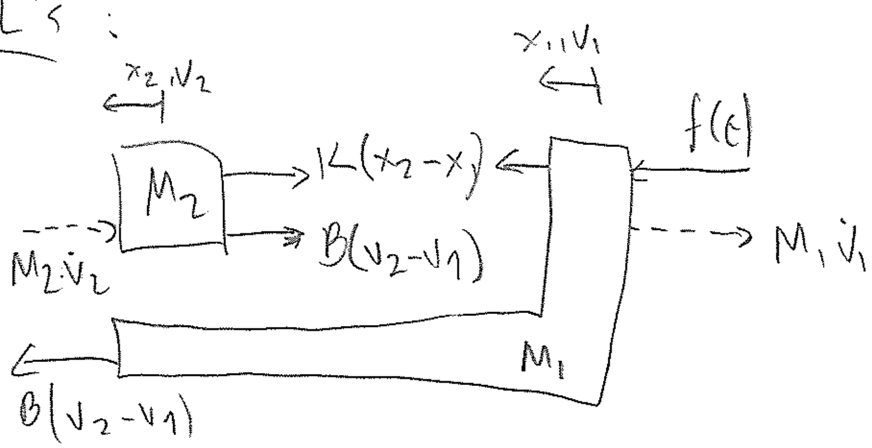
$$\begin{bmatrix} \theta \\ \ddot{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{-K_1}{M_1} & 0 & \frac{K_1 a}{M_1} & -\frac{B_1}{M_1} & 0 & 0 \end{bmatrix} \underline{z} + \begin{bmatrix} 0 \\ 1/M_1 \end{bmatrix} fa(t)$$

3) Obter

$$G(s) = \frac{V_1(s)}{F(s)}$$




DCL's:




$$\sum_i F_i = 0$$

TERMOS:

$M_1$: $-M_2\dot{v}_2 - Kx_2 + Kx_1 - Bv_2 + Bv_1 = 0$ ① $(x_1, x_2)$

$M_2$: $Kx_2 - Kx_1 + Bv_2 - Bv_1 + f - M_1\dot{v}_1 = 0$ ② $(x_1, x_2, f)$

ESTRATÉGIA P/ OBTER G(s):

(I) $TL|_{CI's=0}$ ①, ② ⇒ (1a), (2a)

(II) Isolar $x_2(s)$ em (1a) e subst. em (2a)

⇒ (I) $-M_2 s^2 X_2(s) - K X_2(s) + K X_1(s) - Bs X_2(s) + Bs X_1(s) = 0$

$$X_2(s)\left(M_2 s^2 + Bs + K\right) = (Bs + K) X_1(s) \quad (1a)$$

$$0 = K X_2(s) - K X_1(s) + Bs X_2(s) - Bs X_1(s) + F(s) - M_1 s^2 X_1$$

$$X_2(s)\left(Bs + K\right) - X_1(s)\left(M_1 s^2 + Bs + K\right) + F(s) = 0 \quad 2a$$

II $$X_2(s) = \frac{Bs + K}{M_2 s^2 + Bs + K} X_1(s) \quad 1b$$

1b $\rightarrow$ 2a

$$\frac{(Bs+K)^2}{M_2 s^2 + Bs + K} X_1(s) - X_1(s)\left(M_1 s^2 + Bs + K\right) + F(s) = 0$$

$$X_1(s)\left[-\frac{(Bs+K)^2}{M_2 s^2 + Bs + K} + \left(M_1 s^2 + Bs + K\right)\right] = F(s)$$

$$X_1(s)\left[-K^2 - B^2 s^2 - 2KBs + M_1 M_2 s^4 + M_1 B s^3 + M_1 K s^2 + M_2 B s^3 + B^2 s^2 + KBs + M_2 K s^2 + KBs + K^2\right] =$$

$$\left[M_2 s^2 + Bs + K\right] F(s)$$

$$X_1(s)\left[M_1 M_2 s^4 + (M_1 + M_2) B s^3 + (M_1 + M_2) K s^2\right] = \left[M_2 s^2 + Bs + K\right] F(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + Bs + K}{M_1 M_2 s^4 + (M_1 + M_2) B s^3 + (M_1 + M_2) K s^2}$$

COMO TEMOS $V_1(s) = s X_1(s)$, LOGO

$$\frac{V_1(s)}{F(s)} = \frac{M_2 s^2 + Bs + K}{M_1 M_2 s^3 + (M_1 + M_2) B s^2 + (M_1 + M_2) K s} \quad //$$

$$\frac{V_1(s)}{F(s)} = \frac{\frac{1}{M_1} s^2 + \frac{B}{M_1 M_2} s + \frac{K}{M_1 M_2}}{s^3 + B \frac{M_1 + M_2}{M_1 M_2} s^2 + K \frac{(M_1 + M_2)}{M_1 M_2} s}$$

$$\frac{V_1(s)}{F(s)} = \frac{b_0 s^2 + b_1 s + b_2}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$b_0 = 1/M_1$$

$$b_1 = B/M_1 M_2$$

$$b_2 = K/M_1 M_2$$

$$a_1 = \frac{B(M_1 + M_2)}{M_1 M_2}$$

$$a_2 = \frac{K(M_1 + M_2)}{M_1 M_2}$$

$$a_3 = 0$$